

Quantum magnetotransport properties of topological insulators under strain

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Abstract

Recent experiments reveal that the strained bulk HgTe can be regarded as a three-dimensional topological insulator (TI). Motivated by this, we explore the strain effects on the magnetotransport properties of the HgTe surface states at magnetic field. We analytically derive the zero frequency Hall and collisional conductivities, and find that the substrate induced strain associated with the surface index of carriers, can result in the well separated surface quantum Hall plateaus and Shubnikov-de Haas oscillations. These effects can be used to generate and detect surface polarization.

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1 Introduction

Recently, two- (2D) and three-dimensional (3D) topological insulators (TIs) have drawn much attention in condensed matter physics [1–11]. The relevant researches on 3D TIs mostly focus on Bi_2Te_3 , Bi_2Se_3 , and Sb_2Te_3 compounds, which possess a bulk energy gap and gapless conducting surface states [7–9]. Thereinto, the surface states arise from the mismatch of the bulk topological invariants on two sides of the surface, and can be regarded as the 2D nonideal Dirac fermions with a single Dirac cone at high symmetry points in the first Brillouin zone [7, 8, 10]. These unique properties account for the common features with graphene [1, 3, 12]. However, such 3D TIs have strong defect doping and low carrier mobility in experiment, so that the bulk conductivity always obscures the surface charge transport. Typically, the predicted quantized magnetoelectric effect [13, 14] and the surface Majorana fermions [15], can be found only when bulk carriers are negligible compared to the surface states. Hence, experimentally reaching the intrinsic TI regime, where bulk carriers are absent, is now the central focus of the field.

While so far the focus has been on the above mentioned compounds, recently growing attention is being paid to HgTe quantum wells [16], in which the TI surface states were firstly predicted and observed [1, 2]. Bulk HgTe also has Dirac-like surface states (Refs. 17 and 18) that originate from the inversion between Γ_6 electron and Γ_8 light-hole bands, while the bulk band of Γ_8 heavy-hole coexists with the surface state band, so that the surface states are always coupled by metallic bulk states [17, 18]. This means that 3D HgTe is a semimetal and thus not a TI in the strict sense. With strain applied, a bulk insulating gap (~ 22 meV) opens up at the touching point between the light- and heavy-hole Γ_8 bands, and accordingly the strained bulk HgTe becomes the real TI [3, 18]. Further transport measurements for the strained HgTe on CdTe substrate exhibit the Rashba-like splitting induced by the inversion symmetry breaking in a magnetic field [18, 19]. Also, the Landau levels (LLs) are found to remain degenerate as long as the hybridization can be neglected between the top and bottom surface states (e.g. 70-nm-thick HgTe).

Motivated by this, we have theoretically investigate the strain effects on the Shubnikov–de Haas (SdH) oscillations and Hall plateaus in the zero frequency (dc) collisional and Hall conductivities using the Kubo formalism. These magnetic oscillations appear due to the interplay of the quantum LLs with the Fermi energy, and serve as a powerful technique to investigate the Fermi surface and the spectrum of electron excitations. Our findings show that the substrate induced strain could remove the LLs' surface degeneracy in inversion symmetric

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Dirac cones on the top and bottom surfaces, which is supported by HgTe transport experiments [18,19]. Thus, the Dirac particles of different surfaces present the well separated quantum Hall and SdH effects with different amplitudes and phases. Accordingly, this gives rise to the splitting of LLs and the asymmetric spectrum of the conductivity, in company with the mixture of LLs. Furthermore, we clarify the connections of the SdH and Hall conductivities for different surfaces to the abnormal integer Hall plateaus and SdH beating pattern, and the strain induced changes in the zero-mode conductivity at different surfaces, etc. These phenomena, absent in a conventional 2D electron gas (2DEG) and even in graphene [20–26], should be attributed to the anomalous spectrum of surface states in a fully strained TI. Our results are general and can also be applied to Bi₂Se₃, Sb₂Te₃, and Bi₂Te₃, which would be very great news and certainly meet much interest by the experimental groups.

2 Results and discussion

The surface states moves in the x-y plane under strain (Δ) induced by the substrate, subjected to an external magnetic field $\mathbf{B} = (0, 0, B)$. Here we use the two Dirac cones model to describe the experiments, and the 2D nonideal Dirac quasiparticle Hamiltonian reads

$$H = \tau_z v_F (\sigma_x \pi_y - \sigma_y \pi_x) + \tau_z \mathbf{I} \Delta. \quad (1)$$

In Eq. (1), the first term arises from the spin-orbit coupling (SOC). Due to the strong SOC, the TIs exhibit a unique spin-momentum locking, which is essential for modeling topologically nontrivial insulators. The second term results from the strain energy with \mathbf{I} for the identity matrix and $\tau_z = \pm 1$ for the two surfaces of top (facing vacuum) and bottom (at CdTe interface). The Fermi velocity v_F of surface states in mercury telluride is 4.0×10^5 m/s smaller than that in graphene ($v_F = 1.0 \times 10^6$ m/s), $\{\sigma_x, \sigma_y\}$ is the vector of spin Pauli matrices, and $\pi = \mathbf{p} + e\mathbf{A}/c$ is the canonical momentum with c for the speed of light and \mathbf{A} for the vector potential yielding Landau gauge $\mathbf{A} = (0, Bx, 0)$. The resulting eigenvalues are [18,19]

$$E_{n,\lambda}^{\tau_z} = \begin{cases} \lambda \hbar \omega_c \sqrt{n} + \tau_z \Delta, & n > 0, \\ \tau_z \Delta, & n = 0. \end{cases} \quad (2)$$

with λ for the electron ($\lambda = +1$) and hole ($\lambda = -1$) bands. Integer n ($n = 0, 1, 2, \dots$) represents the LLs ($n = 0, 1, 2, \dots$). The cyclotron frequency is given by $\omega_c = \sqrt{2} v_F / \ell_c$ with $\ell_c = \sqrt{\hbar / eB}$ for the magnetic length. The eigenfunctions of $\Psi_{n,\lambda}^{\tau_z}(r)$ reads

$$\begin{aligned} \Psi_{n,+1}^{\tau_z}(r) &= \frac{\exp(ik_y y)}{\sqrt{L_y}} \begin{pmatrix} \alpha_n \Phi_{n-1}(\xi) \\ -\beta_n \Phi_n(\xi) \end{pmatrix}, \\ \Psi_{n,-1}^{\tau_z}(r) &= \frac{\exp(ik_y y)}{\sqrt{L_y}} \begin{pmatrix} \beta_n \Phi_{n-1}(\xi) \\ \alpha_n \Phi_n(\xi) \end{pmatrix}, \end{aligned} \quad (3)$$

in which $k_y = 2\pi l / L_y$ ($l = 0, 1, 2, \dots$) is the quantum number corresponding to the translation symmetry along the y axis with L_y for the size of the surface in y direction. The prefactors α_n and β_n is, respectively, the cosine and sine of $\Theta/2$ with $\Theta = \arctan \left[\omega_c \sqrt{n/2} / (\tau_z \Delta) \right]$ as

$$\begin{aligned} \alpha_n &= \sqrt{\frac{F_{\lambda n} + \tau_z \Delta}{2F_{\lambda n}}}, \\ \beta_n &= \sqrt{\frac{F_{\lambda n} - \tau_z \Delta}{2F_{\lambda n}}}, \end{aligned} \quad (4)$$

where $F_{\lambda n} = \lambda F_n$ with $F_n = \sqrt{n \hbar^2 \omega_c^2 + \Delta^2}$. The harmonic oscillator eigenfunctions Φ_n are expressed in the normalized Hermitian polynomials $H_n(\xi)$ as $\Phi_n = H_n(\xi) \exp(-\xi^2/2) / \sqrt{2^n n! \ell_c \sqrt{\pi}}$, where we have $\xi = (x + x_c) / \ell_c$ with $x_c = -\ell_c^2 k_y$ for the location of TI states in \mathbf{x} .

Assuming the electrons are elastically scattered by randomly distributed charged impurities, we calculate the dc collisional conductivity following the approaches [27, 28] as

$$\begin{aligned}\sigma_{xx} &= \frac{\beta e^2}{S} \sum_{\zeta, \zeta'} f(E_\zeta) [1 - f(E_{\zeta'})] \\ &\quad \times W_{\zeta\zeta'}(E_\zeta, E_{\zeta'}) (x_\zeta - x_{\zeta'})^2,\end{aligned}\quad (5)$$

where $S = L_x L_y$ and $\beta = \frac{1}{k_B T}$ with k_B for the Boltzmann constant. The Fermi-Dirac distribution function is given by $f(E_\zeta) = [\exp(E_\zeta - \mu)/k_B T + 1]^{-1}$ with the chemical potential μ . The expectation value of $x_\zeta = \langle \zeta | x | \zeta \rangle$ is evaluated as $x_\zeta = \ell_c^2 k_y$ and $x_{\zeta'} = \ell_c^2 k'_y$, so that $(x_\zeta - x_{\zeta'})^2 = \ell_c^4 q_y^2$ due to $k_y = k'_y + q_y$ ($q_y = q \sin \varphi$ and $q^2 = q_x^2 + q_y^2$). Since the scattering is elastic and the eigenvalues do not depend on k_y , only the transitions $n \rightarrow n$ are allowed. Conduction occurs by transitions through spatially separated states from x_ζ to $x_{\zeta'}$, and the transition rate $W_{\zeta\zeta'}(E_\zeta, E_{\zeta'})$ in the presence of impurities reads

$$\begin{aligned}W_{\zeta\zeta'} &= \sum_q |U_q|^2 |\langle \zeta | e^{i\mathbf{q}\cdot\mathbf{r}} | \zeta' \rangle|^2 \\ &\quad \times \delta(E_\zeta - E_{\zeta'}).\end{aligned}\quad (6)$$

Here, $U_q = e^2/2\epsilon\sqrt{q^2 + k_s^2}$ is the Fourier transform of the screened impurity potential $U_r = (e^2/4\pi\epsilon r) \exp(-k_s r)$, where k_s is the screening wave vector; $\epsilon = \epsilon_0 \epsilon_r$ is the dielectric constant. Now we perform an average over random distribution of impurities and denote N_I as the impurity density. By virtue of $\Psi_{n,\lambda}^{\tau_z}(r) \rightarrow |\zeta\rangle$, the $W_{\zeta\zeta'}$ is given by

$$\begin{aligned}W_{\zeta\zeta'} &= \frac{2\pi N_I}{S\hbar} \sum_q |U_q|^2 |\varpi_{n,n'}(\gamma)|^2 \\ &\quad \times \delta(E_\zeta - E_{\zeta'}) \delta_{k_y, k'_y + q_y}\end{aligned}\quad (7)$$

with

$$\varpi_{n,n'}^e(\gamma) = \alpha_n \alpha_{n'} J_{n-1, n'-1}(\gamma) + \beta_n \beta_{n'} J_{n, n'}(\gamma) \quad (8)$$

for electrons and

$$\varpi_{n,n'}^h(\gamma) = \alpha_n \alpha_{n'} J_{n, n'}(\gamma) + \beta_n \beta_{n'} J_{n-1, n'-1}(\gamma), \quad (9)$$

for holes, where $\gamma = l_c^2 q^2/2$. The fuctions $\Psi_{n,\lambda}^{\tau_z}(r)$ oscillating around the point $-x_c$ allows

$$\sum_{k_y} \rightarrow \frac{L_y}{2\pi} \int_{-L_x/2\ell_c^2}^{L_x/2\ell_c^2} dk_y = \frac{S}{2\pi\ell_c^2}, \quad (10)$$

and further using cylindrical coordinates

$$\sum_q \rightarrow \frac{S}{4\pi^2 l_c^2} \int_0^{2\pi} d\varphi \int_0^\infty d\gamma, \quad (11)$$

we can now evaluate (5) with Eqs. (6), (10) and (11) for elastic scattering $f(E_\zeta) = f(E_{\zeta'})$

$$\begin{aligned}\sigma_{xx} &= \frac{N_I \beta e^2 (e^2/2\epsilon)^2}{\hbar \hbar \omega_c 4\pi^2} \sum_{n, \tau_z} f(E_\zeta) [1 - f(E_\zeta)] \\ &\quad \times \int_0^\infty \int_0^{2\pi} \frac{q_y^2 |\varpi_{n,n'}(\gamma)|^2}{q^2 + k_s^2} d\varphi d\gamma.\end{aligned}\quad (12)$$

since $q_y = q \sin \varphi$ and $q^2 = 2\gamma/l_c^2$ as defined above, Eq. (12) reads

$$\begin{aligned}\sigma_{xx} &= \frac{N_I \beta e^2 (e^2/2\epsilon)^2}{\hbar \hbar \omega_c 4\pi} \sum_{n, \tau_z} f(E_\zeta) [1 - f(E_\zeta)] \\ &\quad \times \int_0^\infty \frac{\gamma |\varpi_{n,n'}(\gamma)|^2}{\gamma + \gamma_c} d\gamma.\end{aligned}\quad (13)$$

with $\gamma_e = l_e^2 k_s^2 / 2$. For the small q limit $q \ll k_s$, $(\gamma + \gamma_e)^{-1}$ is expanded in powers of γ/γ_e and we keep the dominant term

$$\begin{aligned} \sigma_{xx} &= \frac{2N_I \beta e^2}{\hbar \hbar \omega_c} \frac{(e^2/2\epsilon)^2}{4\pi l_e^2 k_s^2} \sum_{n, \tau_z} f(E_\zeta) [1 - f(E_\zeta)] \\ &\times \int_0^\infty \gamma |\varpi_{n, n'}(\gamma)|^2 d\gamma. \end{aligned} \quad (14)$$

The calculation of the integral $\int_0^\infty \gamma |\varpi_{n, n'}(\gamma)|^2 d\gamma$ in (14) requires

$$|J_{n, n'}(\gamma)|^2 = \frac{n!}{n!} e^{-\gamma} \gamma^{n'-n} \left[L_{n'}^{n'-n}(\gamma) \right]^2, \quad n \leq n' \quad (15)$$

and the functional relations of Laguerre polynomials [29]:

$$L_n^0(\gamma) = L_n(\gamma), \quad (16)$$

$$L_n^{\alpha-1}(\gamma) = L_n^\alpha(\gamma) - L_{n-1}^\alpha(\gamma), \quad (17)$$

$$x L_n^{\alpha+1}(\gamma) = (n + \alpha + 1) L_n^\alpha(\gamma) - (n + 1) L_{n+1}^\alpha(\gamma), \quad (18)$$

$$\int_0^\infty e^{-\gamma} \gamma^\alpha L_n^\alpha(\gamma) L_m^\alpha(\gamma) d\gamma = \Gamma(\alpha + n + 1) \delta_{nm} / n!. \quad (19)$$

Firstly, making use of (15), one can find some integral identity (e.g. electrons) for $n = n'$:

$$\begin{aligned} \int_0^\infty \gamma [\beta_n^2 J_{n, n}(\gamma)]^2 d\gamma &= \int_0^\infty \gamma [\beta_n^2 J_{n, n}(\gamma)]^2 d\gamma \\ &= \int_0^\infty \beta_n^4 \gamma e^{-\gamma} [L_n(\gamma)]^2 d\gamma \\ &= \int_0^\infty \beta_n^4 e^{-\gamma} [\gamma L_n(\gamma)] L_n(\gamma) d\gamma. \end{aligned} \quad (20)$$

Then, with Eqs. (16) – (18), the term of $\gamma L_n(\gamma)$ can be solved

$$\begin{aligned} \gamma L_n(\gamma) &= n L_n^{-1}(\gamma) - (n + 1) L_{n+1}^{-1}(\gamma) \\ &= n [L_n(\gamma) - L_{n-1}(\gamma)] - (n + 1) \\ &\quad \times [L_{n+1}(\gamma) - L_n(\gamma)], \end{aligned} \quad (21)$$

Further inserting Eqs. (21) into (20), we finally obtain with Eq. (19)

$$\int_0^\infty \gamma [\beta_n^2 J_{n, n}(\gamma)]^2 d\gamma = \beta_n^4 (2n + 1). \quad (22)$$

In like manner, one can derive

$$\int_0^\infty \gamma [\alpha_n^2 J_{n-1, n-1}(\gamma)]^2 d\gamma = \alpha_n^4 (2n - 1) \quad (23)$$

and

$$\int_0^\infty 2\gamma \alpha_n^2 \beta_n^2 J_{n, n}(\gamma) J_{n-1, n-1}(\gamma) d\gamma = -2n \alpha_n^2 \beta_n^2. \quad (24)$$

In a word, since the scattering is elastic and the eigenvalues do not depend on k_y , only the transitions $n \rightarrow n$ are allowed, so that

$$\begin{aligned} \int_0^\infty \gamma |\varpi_{n, n}(\gamma)|^2 d\gamma &= (2n - 1) \alpha_n^4 + (2n + 1) \\ &\quad \times \beta_n^4 - 2n \alpha_n^2 \beta_n^2, \end{aligned} \quad (25)$$

for $n > 0$ and $|\varpi_{0,0}(\gamma)|^2 = e^{-\gamma}$ for $n = 0$. Finally, we get the results, respectively, for electrons

$$\sigma_{xx}^e = \frac{e^2}{h} \frac{N_I \beta e^4}{4\epsilon^2 l_c^2 k_s^2 \hbar \omega_c} \sum_{n, \tau_z} [(2n+1) \beta_n^4 + (2n-1) \alpha_n^4 - 2n \alpha_n^2 \beta_n^2] f(E_\zeta) [1 - f(E_{\zeta'})] \quad (26)$$

and for holes

$$\sigma_{xx}^h = \frac{e^2}{h} \frac{N_I \beta e^4}{4\epsilon^2 l_c^2 k_s^2 \hbar \omega_c} \sum_{n, \tau_z} [(2n-1) \beta_n^4 + (2n+1) \alpha_n^4 - 2n \alpha_n^2 \beta_n^2] f(E_\zeta) [1 - f(E_{\zeta'})] \quad (27)$$

Figure 1 exhibits the collisional conductivity of the top and bottom surface states with the SdH periodicity for zero and finite strain. For null strain, the spectra are perfectly symmetric with a single peak at the CNP confirming the zero-energy TI states (see Fig. 1). This symmetry indicates the surface degeneracy in LLs and thus the SdH oscillations in σ_{xx} are in phase for different surfaces. Whereas, with strain applied, the single peak splits into two ones with a gap opening at the CNP and a well resolved beating pattern of SdH oscillations appears away from the CNP. To clarify these phenomena, we further plot Fig. 2. As seen, the SdH oscillations for different surfaces are out of phase. This demonstrates that the strain breaks the LLs' surfaces degeneracy as well with the inversion symmetry of two Dirac cones at both surfaces, which agrees well with the results of experiments and two Dirac cones model [18, 19]. Here we remark that such a gap ($\Delta_{gap} = 2\Delta$) (see arrow line 1) does not open between the electrons and holes for the single Dirac point in each a surface, but between the two Dirac points at both surfaces. This indicates the two Dirac points at both surfaces shift in the different directions, suggesting the occurrence of two inversion asymmetric Dirac cones.

Further analysis reveals that the two peaks in Fig. 1 for finite strain are the superposition of four peaks in Fig. 2, i.e., top electron (τ_{+1}^e), top hole (τ_{+1}^h), bottom electron (τ_{-1}^e), and bottom hole (τ_{-1}^h). In the same surface, the zero mode peaks do not split, indicating that the Dirac point at each a surface is not gapped. However, for the top electrons and bottom holes, the two peaks do split since the strain lifts the degeneracy of their levels at $n = 0$. Furthermore, for the bottom electrons and top holes, the two peaks not only split but also exchange (see arrow line 2 in Fig. 2), suggesting a mixture of LLs. In a word, the strain removes the LLs' degeneracy in inversion symmetric Dirac cones at both surfaces, and causes the asymmetric conductivity spectrum presenting different amplitudes or different phases or both two ones. Such a lifting of degeneracy usually occurs due to Zeeman coupling in a conventional 2DEG. However, Zeeman coupling here cannot remove the degeneracy since the inversion symmetry is preserved by the magnetic field. And the experiments have already definitely excluded the influence of hybridization between the top and bottom surface states since the width of surface state (2 – 3 nm) is much smaller than the thickness of sample (70-nm) [18, 19]. So the highly possible mechanism for the degeneracy lifting (or the LLs splitting) is the inversion asymmetry due to the strain. Also, from Eq. (2), one can see the surface degeneracy of LLs is removed for any nonzero Δ , which makes the mechanism of inversion-symmetry breaking the most likely explanation for the removed degeneracy. As pointed out in Ref. [18, 19], the strain induced different electrostatic environments of both surfaces, breaks the inversion symmetry of two Dirac cones, lifts the surface degeneracy in LLs, and leads to the asymmetric conductivity for different surfaces.

The dc Hall conductivity σ_{yx} is derived from the nondiagonal elements of the conductivity tensor as

$$\sigma_{yx} = \frac{i\hbar e^2}{S} \sum_{\zeta \neq \zeta'} f(E_\zeta) [1 - f(E_{\zeta'})] \langle \zeta | v_x | \zeta' \rangle \langle \zeta' | v_y | \zeta \rangle \times \frac{1 - \exp\left(\frac{E_\zeta - E_{\zeta'}}{k_B T}\right)}{(E_\zeta - E_{\zeta'})^2}. \quad (28)$$

If we use $f(E_\zeta) [1 - f(E_{\zeta'})] [1 - e^{\beta(E_\zeta - E_{\zeta'})}] = f(E_\zeta) - f(E_{\zeta'})$, Eq. (31) takes the form of the well known

Kubo-Greenwood formula

$$\begin{aligned}\sigma_{yx} &= \frac{i\hbar e^2}{S} \sum_{\zeta \neq \zeta'} [f(E_\zeta) - f(E_{\zeta'})] \\ &\quad \times \frac{\langle \zeta | v_x | \zeta' \rangle \langle \zeta' | v_y | \zeta \rangle}{(E_\zeta - E_{\zeta'})^2}.\end{aligned}\quad (29)$$

since $v_x = \partial H / \partial p_x = -\tau_z v_F \sigma_y$ and $v_y = \partial H / \partial p_y = \tau_z v_F \sigma_x$, we get

$$\langle \zeta | v_x | \zeta' \rangle = i\tau_z v_F (\alpha_n \beta_{n'} \delta_{n-1, n'} - \alpha_{n'} \beta_n \delta_{n, n'-1}) \quad (30)$$

and

$$\langle \zeta' | v_y | \zeta \rangle = \tau_z v_F (\alpha_n \beta_{n'} \delta_{n', n-1} + \alpha_{n'} \beta_n \delta_{n'-1, n}), \quad (31)$$

and thus

$$\begin{aligned}I_{nn'} &= \langle \zeta | v_x | \zeta' \rangle \langle \zeta' | v_y | \zeta \rangle = i v_F^2 (|\alpha_n \beta_{n'}|^2 \\ &\quad \times \delta_{n-1, n'} - |\alpha_{n'} \beta_n|^2 \delta_{n, n'-1})\end{aligned}\quad (32)$$

As usual the matrix elements between the zeroth level and the other levels should be treated separately [30]. Corresponding to Eq. (35) one arrives at

$$I_{0n'} = -i v_F^2 |\alpha_{n'}|^2 \delta_{0, n'-1}, \quad I_{n0} = i v_F^2 |\alpha_n|^2 \delta_{n-1, 0}. \quad (33)$$

In essence, σ_{yx} is the diffusive contribution as the collisional contribution to Hall conductivity vanishes, since the difference of the matrix elements $x_\zeta - x_{\zeta'} = 0$. The following calculations require summing the terms that include all combinations of the matrix elements $\Sigma_{\lambda\lambda'}$, that is, Σ_{++} , Σ_{+-} , Σ_{-+} , Σ_{--} . On the same line [30], we gives $\sigma_{\lambda\lambda} = \Sigma_{++} + \Sigma_{--}$ as

$$\begin{aligned}\sigma_{yx}^{\lambda\lambda} &= 2A \sum_{n, \tau_z} \frac{1}{4F_n F_{n+1} (E_{n,+}^{\tau_z} - E_{n+1,+}^{\tau_z})^2} \{ [f(E_{n,+}^{\tau_z}) \\ &\quad - f(E_{n+1,+}^{\tau_z}) + f(E_{n,-}^{\tau_z}) - f(E_{n+1,-}^{\tau_z})] B \\ &\quad + [f(E_{n,+}^{\tau_z}) - f(E_{n+1,+}^{\tau_z}) + f(E_{n+1,-}^{\tau_z}) \\ &\quad - f(E_{n,-}^{\tau_z})] C \}\end{aligned}\quad (34)$$

and $\sigma_{\lambda\lambda'} = \Sigma_{+-} + \Sigma_{-+}$ as

$$\begin{aligned}\sigma_{yx}^{\lambda\lambda'} &= 2A \sum_{n, \tau_z} \frac{1}{4F_n F_{n+1} (E_{n,+}^{\tau_z} + E_{n+1,+}^{\tau_z})^2} \{ [f(E_{n,+}^{\tau_z}) \\ &\quad - f(E_{n+1,+}^{\tau_z}) + f(E_{n,-}^{\tau_z}) - f(E_{n+1,-}^{\tau_z})] D \\ &\quad + [f(E_{n,-}^{\tau_z}) - f(E_{n+1,+}^{\tau_z}) + f(E_{n+1,-}^{\tau_z}) \\ &\quad - f(E_{n,+}^{\tau_z})] E \}.\end{aligned}\quad (35)$$

Here we define $A = \hbar e^2 v_F^2 / S$, $B = F_n F_{n+1} - \Delta^2$, $C = \tau_z \Delta (F_n - F_{n+1})$, $D = F_n F_{n+1} + \Delta^2$, $E = \tau_z \Delta (F_n + F_{n+1})$. Furthermore, one needs use Eq. (10), which indeed gives the degeneracy for each LL and each valley. Notice that the degeneracy in the zeroth level is only half of that in the nonzero level, so that we have

$$\begin{aligned}\sigma_{yx} &= \sigma_{yx}^{\lambda\lambda} + \sigma_{yx}^{\lambda\lambda'} \\ &= \sigma_{yx}^a + \sigma_{yx}^b + \sigma_{yx}^c\end{aligned}\quad (36)$$

with

$$\begin{aligned}\sigma_{yx}^a &= \frac{e^2}{h} \sum_{n, \tau_z} \left(n + \frac{1}{2} \right) [f(E_{n,+}^{\tau_z}) - \\ &\quad f(E_{n+1,+}^{\tau_z}) + f(E_{n,-}^{\tau_z}) - f(E_{n+1,-}^{\tau_z})],\end{aligned}\quad (37)$$

$$\begin{aligned}\sigma_{yx}^b = & -\frac{e^2\Delta^2}{h} \sum_{n,\tau_z} \frac{\sqrt{n(n+1)}}{F_n F_{n+1}} [f(E_{n,+}^{\tau_z}) \\ & - f(E_{n+1,+}^{\tau_z}) + f(E_{n,-}^{\tau_z}) - f(E_{n+1,-}^{\tau_z})],\end{aligned}\quad (38)$$

and

$$\begin{aligned}\sigma_{yx}^c = & \frac{e^2\tau_z\Delta}{8h} \sum_{n,\tau_z} \left[\frac{(2n+1)}{F_{n+1}} - \frac{2\sqrt{n(n+1)}}{F_n} \right] \\ & \times [f(E_{n+1,-}^{\tau_z}) - f(E_{n+1,+}^{\tau_z})] + \left[\frac{2\sqrt{n(n+1)}}{F_{n+1}} \right. \\ & \left. - \frac{(2n+1)}{F_n} \right] [f(E_{n,+}^{\tau_z}) - f(E_{n,-}^{\tau_z})].\end{aligned}\quad (39)$$

In the limit of zero strain and $T \rightarrow 0$, Eq. (39) can be reduced to $\sigma_{yx} = (2e^2/h)(n + \frac{1}{2})$ with the prefactor 2 resulting from the surface degeneracy. Hall plateaus appear at the filling factor $\pm 1, \pm 3, \pm 5, \dots$, agreeing well with the transport experiments [18, 19].

The collisional and Hall conductivities for different surfaces are both shown in Fig. 3 as a function of the chemical potential for zero and finite strain. As shown, under strain some extra Hall plateaus arise and the steps between plateaus coincide with sharp peaks of the collisional conductivity. Resembling Figs. 1 and 2, this originates from the strain removed surface degeneracy in LLs, so that the density of states could form the different Landau ladders for different surfaces, thereby causing the extra quantum plateaus at even filling factors $0, \pm 2, \pm 4, \pm 6, \pm 8, \dots$, etc. As well known, the Hall conductivity of a single Dirac cone in strainless graphene is given by $\sigma_{yx} = (2n+1)e^2/h$ with the odd filling factors $2n+1$, directly leading to the total filling factors $(4n+2)$ (Ref. [31]). It seems that the new Hall plateaus for a single Dirac cone arise from the extra value $\sigma_{xy} = ve^2/h$ with the filling factors $0, \pm 1, \pm 2, \pm 3, \dots$, etc. In fact, these values are nonexistent, and the abnormal Hall behaviour should be attributed to the phase difference of Hall conductivity for different surfaces induced by the strain.

3 Conclusions

In conclusion, we have investigated the strain effects on the quantum magnetotransport properties for the surface states of TIs at finite temperature and magnetic field. The strain are shown to remove the surface degeneracy of LLs in the two inversion symmetric Dirac cones. Thus, the Dirac particles of different surfaces present the well separated quantum Hall and SdH effects with different amplitudes and phases. This accordingly gives rise to the extra Hall plateaus and the SdH beating pattern away from the CNP. In addition, the SdH conductivity under strain possess two zero-mode peaks around the CNP, while for null strain there is just a single CNP peak. We interpret the two peaks are the superposition of four peaks arising from top electron, top hole, bottom electron, and bottom hole. Further analysis reveals that, in the same surface, the zero mode peaks do not split, indicating the Dirac point at each a surface is not gapped. However, for the top electrons and bottom holes, the two peaks do split since the strain lifts the degeneracy of their levels at $n = 0$. Furthermore, for the bottom electron and top hole, the two peaks not only split but also exchange, suggesting a mixture of LLs. These should be sufficient to well appreciate the experimental results on the quantum magnetotransport of the surface states of HgTe.

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